Efficient Placement of Directional Antennas in Infrastructure-based Wireless Networks

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***Abstract*—Over the past decade, the use of directional antennas has immensely proliferated in wireless networks. Methodically positioning and orienting directional antennas can help reduce the interference while saving energy. In an infrastructure-based wireless network, wireless devices communicate through base stations. In this setting, directional antennas placed on the base stations form the backbone of the communication network. The wireless devices distributed around the base stations have bandwidth requirements that have to be satisfied by the directional antennas, and antennas constrained by hardware limits can only serve a limited number of devices. This introduces an optimization problem and in this paper, we develop an integer linear program for placing and directing antennas on multiple base stations to minimize the number of antennas required to serve all the wireless bandwidth demands. Through this integer programming formulation, we analyze the performance of directional antennas under various settings. A simulation-based evaluation using Cplex's branch-and-bound algorithm demonstrates the efficacy of our approach and provides us further in insights into these problems.**

I. INTRODUCTION

The rapid adoption and deployment of wireless devices has put an increasing demand on the bandwidth, speed, and capacity of wireless networks. Advances in antenna technology and planning play an important role in addressing these challenges. Traditional omni-directional antennas uniformly radiate power and can waste power in the areas where there are no wireless devices. Importantly, it has been shown that omni-directional antenna do not scale with the number of wireless devices [5]. Directional antennas [6] are an alternative, promising technology in wireless networks. Directional antennas have various advantages over omni-directional antenna [6], [13]. A directional antenna has focused span of transmission which results in improved network capacity and power efficiency, as well as reduced interference [3].

Over the past few years, researchers have studied many different aspects of directional antennas [7], [17], [12], [1]. Network capacity with directional antennas was studied in [1], [19], [14] and it was demonstrated wireless networks employing directional antennas have better network capacity than networks employing omni-directional antennas. To utilize this increased capacity provided by directional antennas, Choudhury *et al.* designed an efficient medium access control protocol for ad hoc networks and Jawhar *et al.* [7] designed an efficient bandwidth reservation protocol for routing in wireless mesh networks. Together with these algorithmic developments, the advances in hardware aspects of designing steerable antenna [9], [1] have made large-scale deployment of directional antennas feasible.

Our focus in this paper is in optimally aligning directional antennas in infrastructure-based wireless networks such as cellular networks and IEEE 802.11WLANs. In these networks, wireless devices affiliate themselves with base stations, each of which typically has a high-speed connection to the rest of the network. We assume that each subscriber can be only assigned to one antenna (and correspondingly, a single channel). A directional antenna can only provide connectivity to a subscriber if it geometrically covers it. One challenge in using directional antennas is how to geometrically align the antennas to achieve optimal performance. To overcome this challenge, probabilistic algorithms were used to discover neighboring nodes [18], analytical models were made for connecting circular sectors to base stations [19], [15], [18], and graph coloring was used to assign interference free channels [10], [4]. A recent paper [8] investigated various bandwidth provisioning problems from the standpoint of both subscribers (wireless devices) and providers (base stations). In [2], approximation algorithms were developed to find the minimum number of directional antennas needed on a *single base station* to serve all wireless devices. Minimizing the number of antennas is necessary from a network operator's perspective as lesser number of antennas translates into lesser cost for the operator.

Different from the above papers, we consider *multiple base stations* surrounded by wireless devices. This is more realistic scenario as usually network operators have multiple base stations to achieve QoS guarantees. We assume that the locations of these base stations are fixed and given as input to the problem. Each wireless device has its individual bandwidth demand. The goal is to place and orient the directional antennas on base stations to serve all the bandwidth demands while minimizing the number of antennas used. We propose an integer linear programming (ILP) formulation for this problem. We also enhance the model to capture hardware limitations by adding cardinality limits on the maximum number of devices that could be served by an antenna. As the problem turns out to be **NP**-complete, the optimal solution of the ILP cannot always be found in polynomial time (unless **P**=**NP**). We use the ILP to investigate the relationship among key parameters like the number of base stations, variations of bandwidth, and the cardinality constraints. We use Cplex's branch-and -bound algorithm to solve our ILP models

Our paper is organized as follows. In Section II, we will describe the problem and introduce the basic ILP model as well as its variation to include the cardinality limit. In Section III, we describe the simulation setting and discuss the results and their implications. We conclude our paper in Section IV with future directions.

II. DEFINITION AND MODELS

In this section, an ILP formulation is developed to place directional antennas on base stations, to pick the antenna directions, and to assign wireless devices to antennas.

A. *Problem Description*

We first describe the wireless network model and the setting of our problem. Our model is a generalization to the model used in the earlier works of directional antennas [18], [2], [8]. Let J be the set of wireless devices and I be the set of base stations. Similar to [19], [18], [15], [2], [8], we model a directional antenna as a circular sector centered at the base station. Each antenna is assumed to have a bandwidth capacity of b bits/second and a transmission range of R meters. If the Euclidean distance between device j and base station i is smaller than R meters, then device j is reachable from base station i. To deal with directions, we divide a circle to a set L of sectors with an equal central angle (in degree), and each sector represents a direction. With a central angle α, there are $|L| = \lceil 360/\alpha \rceil$ sectors (see Fig. 1) around a base station. The relative direction of a wireless device to a base station is specified by the sector in which the device is located on the base station. Therefore, a wireless device may be in the range to multiple base stations, but at each station the device is located within at most one sector of that base station. Each antenna has a coverage range θ which is measured in the unit of sectors. In other words, if an antenna has a coverage range θ, it can cover contiguous θ sectors. In Fig. 2, an antenna has coverage 3, and two antennas overlap in one sector. The bandwidth requirement for the wireless devices is a $|J|$-tuple $B = \{b_1, \cdots, b_{|J|}\}$. We assume that $\max_{i \in J}\{b_j\} \leq b$, and we normalize all bandwidths by b (i.e., we set antenna capacity to 1 and divide each device bandwidth by b). Henceforth, $0 \leq b_j \leq 1$ is the normalized bandwidth for device j. Device j can be assigned to antenna k, if k is oriented to cover the device (i.e., to cover the sector where j is located). Each device can be only served by one antenna. An antenna can serve multiple devices with requirement that the total bandwidth served (sum of bandwidth of the devices assigned to the antenna) by the antenna is less than equal to 1. The goal of the antenna location problem is to determine the minimum number of antennas to serve all the wireless devices. Below, we summarize the notation used in this paper.

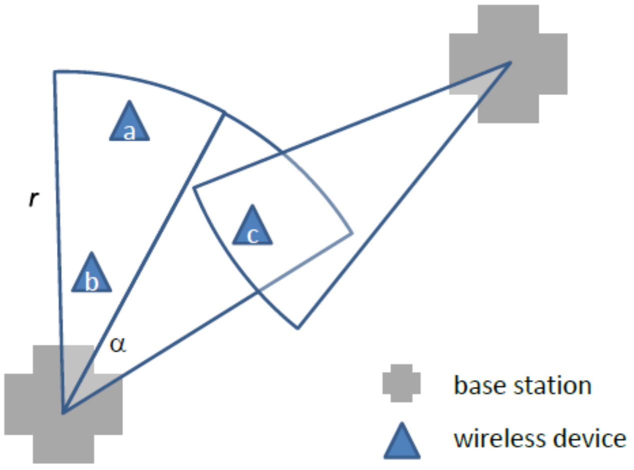

Fig. 1. Base stations, wireless devices, central angle, sectors, and their relation.

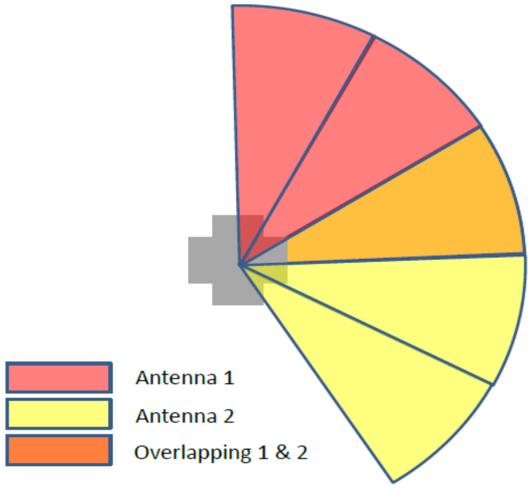

Fig. 2. Antenna and its coverage. Each antenna covers 3 sectors with an overlap on 1 sector.

NOTATION

Index/Set

I	set of base stations
J	set of wireless devices
K	set of antennas
L	set of sectors

Data Parameters

m_{jil}	1 if wireless device j is located at sector l of base station i; 0 otherwise
b_j	bandwidth of wireless device j
θ	angle span of antenna in the unit of sector
c	maximum number of devices that could be served simultaneously by an antenna

Variables for ILP

x_{jk}	0-1 variable set to if wireless device j is served by antenna k; 0 otherwise
y_{kil}	0-1 variable set to 1 if antenna k is located on base station i and covers sectors l to $(l + \theta) \bmod |L|$; 0 otherwise

We assume in this paper that wireless devices are *static* (or move infrequently). This assumption occurs quite common in real-world situations (for example, when providing network connectivity to users in offices, university buildings, airports, etc). Given the locations of these wireless users it would be desirable for the base stations to align its antennas in order to satisfy their bandwidth requirements while minimizing the number of antennas used.

B. Integer Linear Programming Formulation

Next, we formulate the problem described in the previous section as an integer linear program.

There are two sets of variables in our formulation. The assignment between wireless device j and antenna k is denoted as x_{jk}. Variable $x_{jk} = 1$ if antenna k serves[1] device j, and 0 otherwise. The location and direction of antenna k is represented by y_{kil}. Variable $y_{kil} = 1$ if antenna k is placed on station i and it covers the contiguous sectors clockwise from l to $l + \theta$, and 0 otherwise. The addition and subtraction of sector indices are modulo $|L|$ since $\alpha|L| = 360$. The modulo operation will not explicitly expressed in this paper for notation simplicity.

To model the fact that each device can be assigned to only one antenna, we introduce the following set of constraints:

$$\sum_k x_{jk} = 1, \ \forall j \in J. \tag{1}$$

For each antenna, it can serve a device only if the antenna geographically covers the location where the device is located. Let $I(j) = \{i \in I : \sum_l m_{jil} = 1\}$ be the set of neighboring base stations within the distance R from device j where $m_{jil} = 1$ if wireless device j is located at sector l of base station i, and 0 otherwise. For $i \in I(j)$, we use the label l_{ji} for the sector where device j is located in base station i. Again, for each $i \in I(j)$, there is at most one l where $m_{jil} = 1$. It leads to the next set of constraints which make sure that the assignment between device j and antenna k is realizable,

$$x_{jk} \leq \sum_{i \in I(j)} \sum_{l_{ji} - \theta \leq l \leq l_{ji}} y_{kil}, \ \forall j \in J, \forall k \in K. \tag{2}$$

Note the sectors between $l_{ji} - \theta$ to l_{ji} are where antenna k must be located on station i to cover device j, by the clockwise convention. If antenna k is not located and oriented in a sector which can serve device j, the right-hand side of (2) becomes 0. It eliminates the assignment between device j and antenna k by forcing variable x_{jk} to be 0.

There are two constraints for each antenna. One is to enforce that the total bandwidth of the wireless devices served by an antenna is under the normalized antenna capacity of 1,

$$\sum_j b_j x_{jk} \leq 1, \ \forall k \in K. \tag{3}$$

The other constraint ensures that each antenna can be located and oriented at most once

$$\sum_{i,l} y_{kil} \leq 1, \ \forall k \in K. \tag{4}$$

With objective being to minimize the number of used antennas, the resulting antenna location problem (**ALP**) is

$$\begin{aligned} \min_{x,y} \quad & \sum_{i,k,l} y_{kil} & (5) \\ \text{subject to} \quad & (1) - (4), & \\ & y_{kil} \in \{0,1\} \ \forall k \in K, \forall i \in I, \forall l \in L, & \\ & x_{jk} \in \{0,1\} \ \forall j \in J, \forall k \in K. & \end{aligned}$$

In practice, directional antennas have an upper limit on the number of devices that could simultaneously be served. This comes because of the hardware limitations in directional antennas and if too many devices connect to a directional antenna the performance will be very poor [16]. In this paper, we model the maximum number of devices that an antenna could simultaneously serve as an additional constraint on the antenna. Let c be this number. The resulting *cardinality limit* is that an antenna can simultaneously serve at most c devices. This is captured by the following

$$\sum_j x_{jk} \leq c, \ \forall k \in K. \tag{6}$$

After adding these cardinality constraints (6) to the optimization (5), we call the resulting ILP **ALP-C** (standing for antenna location problem with cardinality limit).

Placing the minimum number of antennas to satisfy all bandwidth demands is computationally hard. Kasiviswanathan *et al.* [8] proved that minimum antenna problem with a single base station is **NP**-hard by showing that it's a variation of bin packing, a classical **NP**-hard problem. Since **ALP** is a generalization that allows more than one base station, it is also **NP**-hard.

Theorem 1. **ALP** *is* **NP**-*hard.*

Again, **ALP** is a special case of **ALP-C** obtained by letting the cardinality limit c be a large number such that constraints (6) are never tight. Therefore, **ALP-C** is also a computationally hard problem.

Corollary 1. **ALP-C** *is* **NP**-*hard.*

III. SIMULATION AND CASE STUDY

In this section, we use **ALP** and **ALP-C** to investigate the minimum number of directional antennas required under various settings, which include varying the bandwidth of wireless devices, the number of base stations, and the cardinality limits. One difficulty of using the ILP approach for **ALP** or **ALP-C** is the problem of *symmetry* in the optimal solution [11]. Given an optimal placement of antennas, by symmetry, any permutation of the antenna placement is also optimal since the antenna properties (i.e., coverage range, bandwidth, and cardinality limit) are the same for all antennas. The theoretical algorithm development to overcome the symmetry and to reduce the computation time is beyond the scope of this paper. In this paper, Cplex's branch-and-bound algorithm is used to solve the ILP. Branch-and-bound algorithm is a general algorithm

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[1] That is antenna k assigns a bandwidth of b_j from its capacity of 1 to satisfy the demand of device j.

for solving ILP by repeatedly solving linear programming relaxation of the original problem with enumeration of variables being fixed at integral values.

First, we describe the general setting of the inputs to the ILP models. Later in this section, we discuss the results of individual test cases. To simulate the infrastructure-based wireless network, we place fifty wireless devices (uniformly and independently) at random in a unit square. The normalized bandwidth of each device is generated according to a uniform distribution, Uniform(a, b). The various combinations of a and b will be discussed later as the part of the case studies. Throughout the experiments, the central angle of a sector is set to $20°$, and each antenna can cover 3 contiguous sectors. The transmission range (R) of an antenna is set to $\sqrt{2}/2 = 1/\sqrt{2}$ such that a single antenna placed at $(0.5, 0.5)$ can cover any device in the unit square.

With the above setting and the locations of base stations, the map m_{jil} between device j and base sector (i, l) can be calculated. Then, the neighboring base stations $I(j)$ of device j and the sector location of j in base i (l_{ji}) are derived from m_{jil}. With the above inputs, these ILP models are then implemented in C++ with Cplex 11.0 as the integer programming solver. The results from the experiments are discussed next. We consider two scenarios.

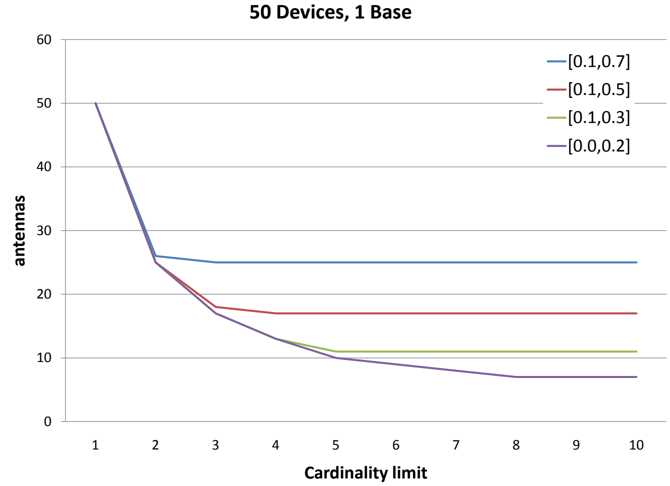

Fig. 3. Single base station case: The plot shows the number of antennas required for various lower and upper bounds on the device bandwidths.

a) Single Base Station: A single base station is placed at the center coordinated $(0.5, 0.5)$. We assume that the lower left corner of the unit square is the origin. With the transmission range being $1/\sqrt{2}$, the base station can connect to any device in the unit square. We generate the bandwidths of the wireless devices independently from the uniform distribution Uniform(a, b). We experiment with different sets of lower (a) and upper bound (b) to test the minimum number of antennas required. As noted earlier, **ALP** is just a special case of **ALP-C**, and therefore, we use **ALP-C** to compute **ALP**. To do so, we set the cardinality limit to a large number. **ALP-C** is then tested by picking various smaller cardinality limits. The results are shown in Fig. 3. In this paper (due to space limitations), we only show results for **ALP-C**.

As we can see the optimal number of required antenna is approximately $\max\{\lceil 50\bar{b}\rceil, 50/c\}$ where $\bar{b}$ is the average bandwidth ($(a+b)/2$) and c is the cardinality limit. The reason for this is that the devices are uniformly placed in the unit square and the bandwidths are generated independently. On average, the number of devices can be served by an antenna depends on which of the constraint (3) and (6) is tight and is approximately the smaller of $\lceil 1/\bar{b}\rceil$ and c. Along the same line of reasoning, in the setting where the device bandwidths generated are between $a = 0.1$ and $b = 0.7$, increasing the cardinality limit above 3 didn't further reduce the minimum number of required antennas.

b) Multiple Base Stations: We now test the effect of multiple base stations. We keep the rest of the setting as in the single base station case except that the number of base stations is now greater than 1.

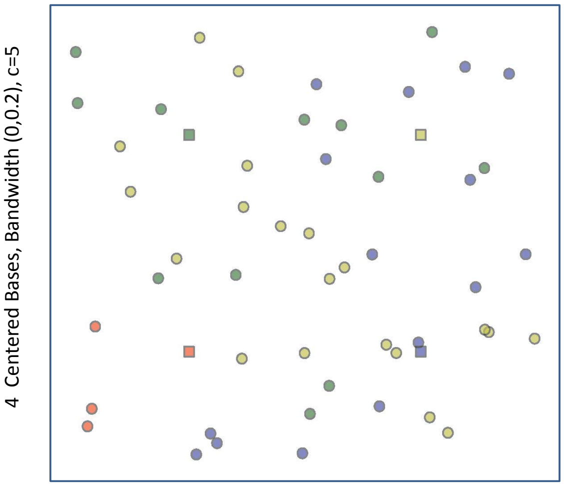

Fig. 4. Assignment of devices to base stations by solving ILP: Base stations are denoted by squares and the devices are denoted by circles. The case is for 4 centered base stations, $a = 0.0$, $b = 0.2$, and $c = 5$. A device is assigned to a base station with the same color.

The four base stations are first placed at the centers of four equally divided sub grids. The four centers are $(0.25, 0.25)$, $(0.25, 0.75)$, $(0.75, 0.25)$, and $(0.75, 0.75)$. Figs. 4 and 5 show the assignment of devices to base stations for bandwidth chosen uniformly between $[0, 0, 2]$ and $c = 5, 10$. Shown in these figures, the ILP solution does not always assign devices to their closest base station. This has to do with the geometry of directional antenna coverage. The experiment results are summarized in Fig. 6. With device bandwidths ranging between $a = 0.1$ and $b = 0.7$, the four-base station case requires 1 less antenna for $c > 3$. However, somewhat surprisingly in the case of bandwidth range between $a = 0$ and $b = 0.2$, it requires 1 more antenna for $c \geq 5$. The main reason for requiring more antennas in this scenario has also to do with the geometry which limits the number of devices

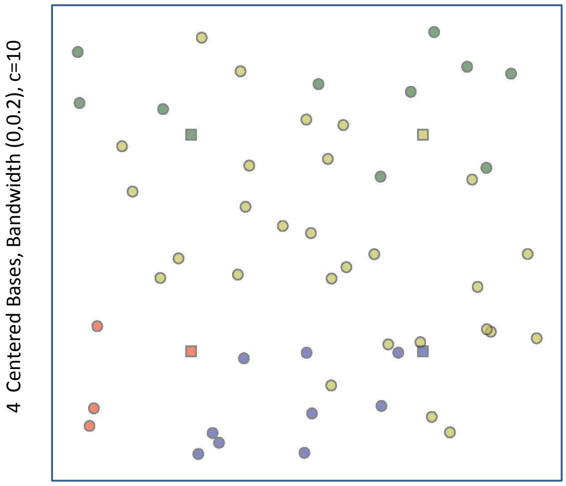

Fig. 5. Assignment of devices to base stations by solving ILP: Base stations are denoted by squares and the devices are denoted by circles. The case is for 4 centered base stations, $a = 0.0$, $b = 0.2$, and $c = 10$. A device is assigned to a base station with the same color.

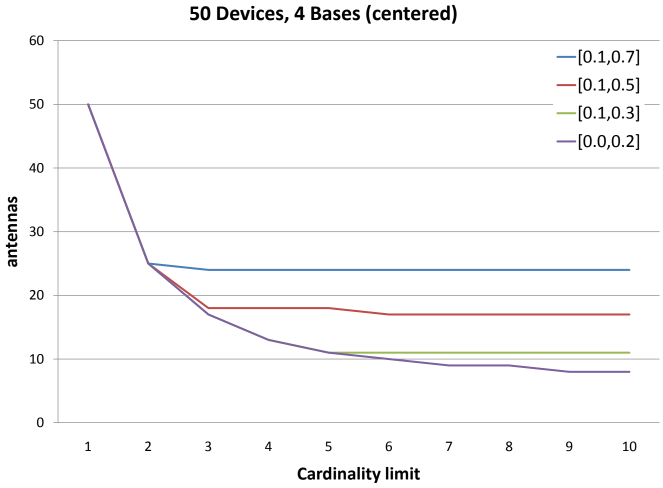

Fig. 6. Multiple base station case: The four base stations are located at the centers of equal sized sub-grids of the unit square. The plot shows the number of required antennas for various lower and upper bounds on the device bandwidths.

which can be covered by a single antenna. With lower device bandwidths, an antenna can serve more devices. Since an antenna has sectoral coverage, the further the device is away from the base station, the more devices that can be packed in one sector (antennas have smaller area coverage closer to the base station, see Fig. 1). In the case of multiple base stations, the distances between devices and stations are shorter, and as a result packing of devices into sectors is sparser. For example, in the single base station case two devices may be in the same sector or neighboring sectors. But, they may be in two or non-adjacent sectors in the case of multiple base stations. Therefore, both bandwidth and cardinality limit of an antenna may not be fully utilized. This problem may be overcome by choosing different locations for the base stations as we do for our next set of experiments.

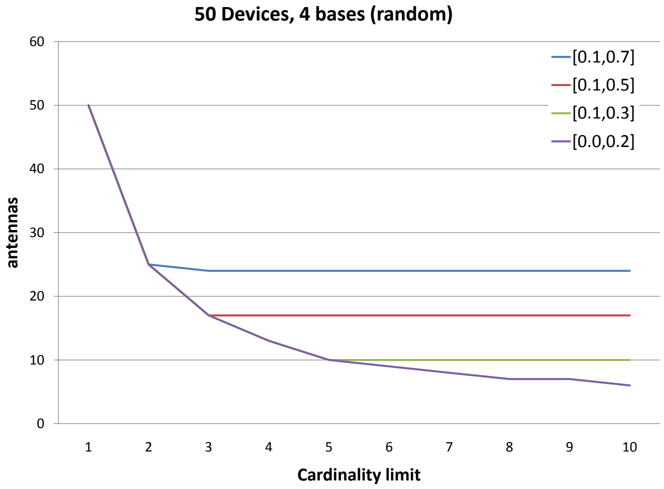

Fig. 7. Multiple base station case: The three base stations are randomly located and one is placed at $(0.5, 0, 5)$. The plot shows the number of required antennas for various lower and upper bounds on the device bandwidths.

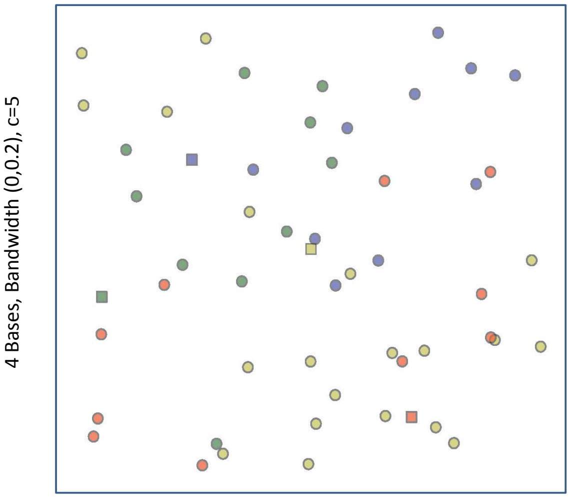

Fig. 8. Assignment of devices to base stations by solving ILP: Base stations are denoted by squares and the devices are denoted by circles. The case is for 1 centered base station and 3 randomly placed base stations, $a = 0.0$, $b = 0.2$, and $c = 5$. A device is assigned to a base station with the same color.

We experimented with the scenario where one station is placed at the center $(0.5, 0.5)$ and the remaining three are randomly placed. The result is shown in Fig. 7. In this placement, the minimum number of required antennas should be no larger than the number in 1 base station case. In fact, the random placement outperforms both the above cases. For example, in the bandwidth interval $[0.1, 0.3]$ and $[0, 0.2]$, one less antenna is required with the cardinality limit being large enough. These results indicate that the location of base stations is an important factor in minimizing the number of antennas and should be added to the antenna location models. However,

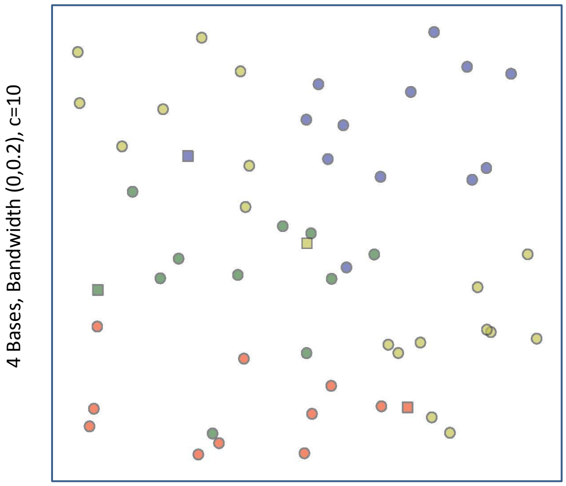

Fig. 9. Assignment of devices to base stations by solving ILP: Base stations are denoted by squares and the devices are denoted by circles. The case is for 1 centered base station and 3 randomly placed base stations, $a = 0.0$, $b = 0.2$, and $c = 10$. A device is assigned to a base station with the same color.

it is not a trivial modeling extension given that it is the continuous space where the base stations can be placed. After a discretization of the space an ILP model can be developed but the computational time can be much longer without some new algorithmic development. In Figs. 8 and 9, we show the assignment between devices and base stations for few values of a, b, c. Again one notices that devices are not always assigned to their closest base station.

IV. CONCLUSION

In this paper, we introduced ILP models for placing minimal directional antennas to serve bandwidth requirements of wireless devices. This problem is **NP**-hard. Using Cplex's branch-and-bound algorithm, we solved these models with different combination of parameters like device bandwidth requirements, number of base stations, and cardinality limits. The results show interesting connections between these parameters that a wireless network designer could exploit to design better and cheaper networks.

Our research also opens up a number of directions for future research. First, in practice wireless networks divide available bandwidth into (possibly overlapping) channels. For example, while there are 11 overlapping channels in IEEE 802.11b only three channels are non-overlapping. Orienting multiple antennas using the same channel such that the regions covered by them overlap may cause degradation of signal quality and throughput. Adding channel constraints to our ILP model would be a natural extension to our model. Second, in this paper, we considered a setting where the wireless devices are static or move very infrequently. This allows our algorithms to be re-executed every time a device moves. However, in highly mobile environments such an approach is clearly impractical. Efficient antenna positioning algorithms for these settings is an extremely interesting direction.

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